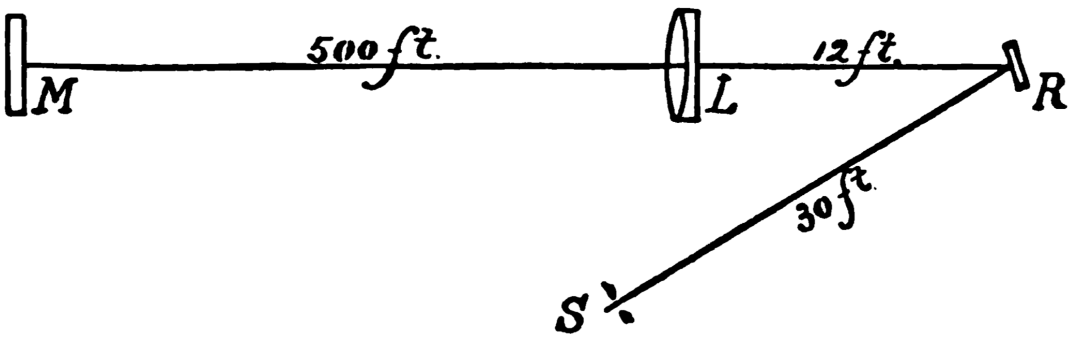
Re-Exam : Waves & Optics

12 April 2018, 14:00-17:00 (Martiniplaza)

- Put your name and student number on each answer sheet.
- Answer all questions short and to the point, but complete; write legible.
- Final point grade = 9(total number of points/39) + 1

1. **Speed of Light (15 points)**
consider the paper *Experimental Determination of the Velocity of Light* by Michelson. Fig. 1 of the paper (and the figure below) gives a schematic overview of the rotating mirror setup used to measure the velocity of light (see below for an short explanation).

Fig. 1.



Sunlight is admitted to the setup through a slit S and aimed at a rotating mirror R. This mirror is placed in the focal point of a lens L, which creates a parallel beam which is reflected from a mirror M. The reflected beam is focused onto the rotating mirror and reflected towards the slit. For a known revolution rate and fixed speed of light the reflected beam will be (slightly) rotated with respect to the incoming beam. By observing the displacement of the reflected beam with respect to the slit the velocity of light can be deduced.

a) Assume that the light emerges and converges at a single point on R. Someone claims the following: *Light rays that pass through the outer edges of lens L travel a longer path than those traveling through the center. Because of this we measure a speed of light that is too small.* Explain whether this argument is correct. **(3 points)**
By construction the rays arriving at a focus have the same phase because they are on a wavefront. Therefore they time needed to travel from R to M and back must be the same as well.

b) Assume the lens has a thickness of 4 inches in the middle and a refractive index of 1.5. If you ignore that the lens is made out of glass, by what fraction does the measured speed of light differ from the true speed of light in air? Note: 1 ft = 12 inches. **(3 points)**
Here we have to consider the optical path length, which is 512 ft + 1.5 times 4 inch = 512*12 + 6 = 6160 inch. This is to be compared to the geometrical pathlength, which is 2 inch (0.5 times 4 inch) shorter. So the measured speed will be about 1/3 permille too low.

c) Lens L is placed air, causing some of the light to be reflected from its entrance surface. Using reasonable material properties, and assuming perpendicular incidence on the lens, calculate the irradiance of the reflected light relative to that of the incident irradiance. **(3 points)**
The answer is obtained using the Fresnel equations, which give the reflection and transmission coefficients for both polarisation orientations. For perpendicular incidence $\theta_{in} = \theta_{out} = 0°$, both orientations are identical. For this we find that $R_s = R_p = R = \left|\frac{n_1 - n_2}{n_1 + n+2}\right|^2$. Note that we need $R = r^2$ and not r, which is the amplitude transmission coefficient. Reasonable values are $n_1 = n_{air} = 1$ and $n_2 = n_{glass} = 1.5$.

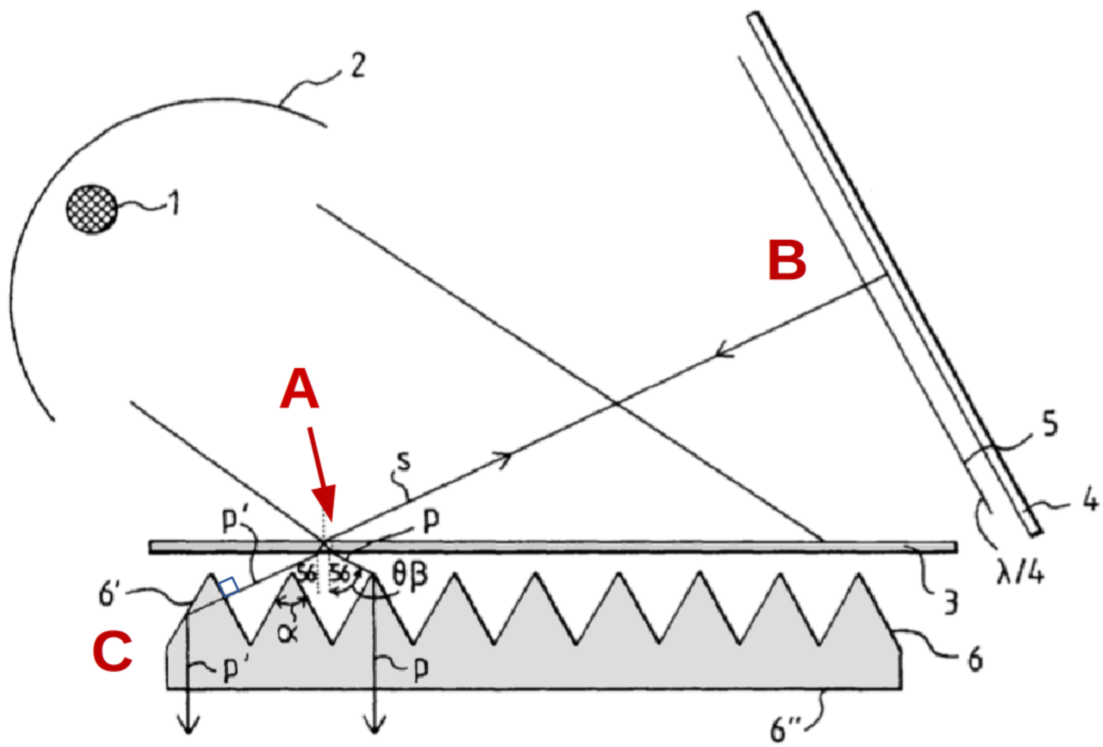
d) A thin layer of material with a refractive index *between* air and glass can be used to create an anti-reflection coating on the surface of the lens. Explain the principle behind this by giving the relevant conditions and formula(s). Is it possible to have no reflection at all? If so, explain under what conditions. If not, explain what prevents it. Tip: carefully consider the phases of the outgoing waves and the difference with a thin film placed in air. Ignore reflections from the back of the lens. **(3 points)**

Relates to section 9.6 of Hecht. The reflections from the first and second surface of the film should interfere destructively. Both are external reflections (optically thin to thicker), so no phase flip occurs. Light reflecting from the entrance surface So an (optical) thickness of $d = 1/2(n + 1/2)\lambda$ would lead to optimal phase difference. If we only consider the ...

e) Assume we place a small pin-hole with a diameter of 1.22 mm at the location of the lens. Because of the limited size a diffraction pattern will appear on the mirror **M**. Would you speak of Fraunhofer or Fresnel diffraction? Why? Calculate the radius of the first dark ring projected onto the mirror, for light with a wavelength of 600 nm. **(3 points)**

See chapter 10.2.5: $q_1 = 1.22R\lambda/2a = 1.22 * 500 * 0.3/1.22x10^{-3} * 600x10^{-9} = 9\,cm$

2. **Polarizer (12 points)**
Consider the United States Patent 6,072,628. *Optical Polarisation Device*
In this patent the invention of a polarized light source is described. In it you find figure 1, which gives the basic setup of a light polarization device (see figure below). Light impinges on a thin glass slab “3” at point **A**. Part of the light is transmitted along path p, and the remainer is reflected to point **B**. Here $\lambda/4$ plate “5” is placed right in front of a mirror “4”. The reflected light again strikes the glass slab and is now transmitted. Then it enters a toothed piece of glass “6” that bends the light along path p.



a) Explain the process occuring at point **A** that causes the light to be polarized. How is the light reflected toward point **B** polarized? The exit-angle is listed as 56 degrees; use this to calculate the refractive index of the glass sheet labeled “3”. **(3 points)**
Full polarization happens at the Brewster angle, where the polarization component perpendicular to the plane of incidence is fully reflected, whereas the parallel component is fully transmitted. The Brewster's angle is given by $\tan\theta_B = n_2/n_1$. Since $\theta_B = 56°$, we find that $n_{glass} = \tan 56° \simeq 1.48$.

b) What is the angle of the beam of light *inside* the glass sheet “3” (relative to the normal)? If you could not solve a), use a reasonable assumption for the refractive index. **(3 points)**
This requires an application of Snell's law: $\sin\theta_1/\sin\theta_2 = n_2/n_1$. So $\theta_2 = \arcsin(\sin\theta_1)/n_1) \simeq 34°$.

c) Which physical phenomena is behind the working of the $\lambda/4$ plate “5”? Explain how the plate should be placed relative to the plane of incidence so that the light reflected by “4” is transmitted at point **A**. **(3 points)**
The physical phenomenon is birefringence, i.e. the phenomenon that the two orthogonal polarization components of the light experience a different refractive index. To rotate the plane of polarization by 90 degrees, the plate should be placed under an angle of 45 degrees

relative to the polarization plane.

d) Use the characteristics of the reflection near point **C** (note: there is no reflective coating applied to the surface) to derive the condition for the refractive index of part "6". Tip: it is possible to find the angle of incidence near **C** without doing a calculation. **(3 points)**
This is a case of total internal reflection. From symmetry arguments it is easily understood that the angle of incidence is 56 degrees. This angle has to be larger than the critical angle, given by $\sin\theta_{cr} = 1/n$. So $n > 1/\sin 56^\circ \simeq 1.21$.

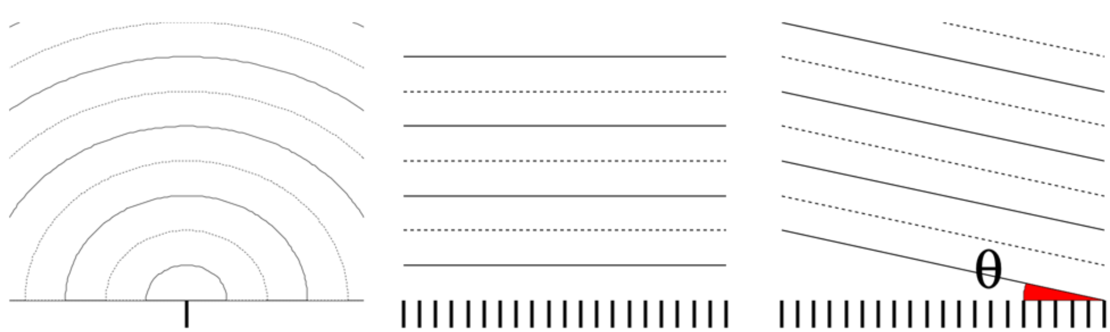
3. **Fiber communication (12 points)**
The use of optical fibers is becoming more and more important for transporting data. Here we will investigate some of the properties.

a) Optical fibers are optimized to have as little dispersion as possible. Explain what "dispersion" is, and why different materials have different dispersion properties. Which material properties are essential? **(3 points)**
Dispersion is the phenomenon that the refractive index is wavelength dependent. The degree of dispersion depends on the damping factor and resonance frequencies of the material. See section 3.5.1.

b) A laser is used to send a very short burst of light into an optical fiber that exhibits dispersion. Explain why dispersion gives rise to a change in the time dependence (shape) of the irradiance of this light burst, in particular when it travels a long distance in the fiber. **(3 points)**
Because of the Fourier theorem, a very short burst of light comprises a broad frequency range. Each frequency will travel at another speed, hence the envelope of the pulse will become time and/or distance dependent.

c) Low-quality optical fibers are made of glass that contains very small contaminants. Explain why these fibers can still be used for transporting red light fairly efficiently, but not for blue light. **(3 points)**
Transport of light in this fiber will suffer from Rayleigh scattering, which is proportional to $1/\lambda^4$. Hence red light, which has a longer waves length than blue light, experiences less scattering than blue light does.

d) Light that exits the fiber is essentially emitted as a spherical wave. When a bundle of fibers is illuminated by the same (coherent) source, and when the fiber-ends are placed on a line, a cylindrical wave is created that emerges from this line-array. The axis of this cylinder is located on the fiber ends. Explain what you would need to do to "steer" the axis of the cylindrical wave (*i.e.* let the axis make an angle $\theta \neq 0$ w.r.t. the line of fiber ends) while keeping the fiber ends at their original locations. You may ignore effects from the ends of the array. Would you get the same result if each fiber were illuminated by it's own light source? The figure illustrates the various configurations.



You would have to manipulate the phases of the light being emitted from the fibers. This can be done by changing the lengths of the fibers, or by changing their refractive index. For a cylinder wave to radiate away under an angle θ, the phase at location z along the array must vary as $\Delta\phi(z) = \frac{z}{\lambda}\tan\theta$ (with modulo 2π as a possibility).